Homework 4 Key

Ch 4   4, 6, 14, 16, 20, 24, 26, 32, 36, 38, 54, 60

(4) Frequency $= 2.83 \times 10^{20} s^{-1}$ (Gamma rays)

Calculate the wavelength in meters & angstroms.

$C = \lambda \cdot V$

$$\lambda = \frac{c}{V} = \frac{2.9979 \times 10^{8} MS^{-1}}{2.83 \times 10^{20} s^{-1}} = \boxed{1.06 \times 10^{-12} M}$$

$10^{-10} M = 1 Å$

$\boxed{0.0106 Å}$

(6) Argon laser $\lambda = 488 nM$

A. Calculate frequency

$$V = \frac{c}{\lambda} = \frac{2.9979 \times 10^{8} MS^{-1}}{(488 \times 10^{-9} M)} = \boxed{6.14 \times 10^{14} s^{-1}}$$

B. Calculate elapsed time for round trip.

$3.8 \times 10^{5} KM$ Earth to Moon

$$\frac{2 \cdot (3.8 \times 10^{8} \cancel{M})}{2.9979 \times 10^{8} \cancel{M} s^{-1}} = \boxed{2.53 s}$$ for the round trip

14 Energy transition $= 4.9 \times 10^{-19}$ J decrease

Calculate the λ emitted and Predict the Color

$$V = \frac{\Delta E}{h} \quad C = \lambda \cdot V \quad C = \frac{\lambda \Delta E}{h} \quad \lambda = \frac{Ch}{\Delta E}$$

$h = 6.62608 \times 10^{-34}$ J s

$$4.05 \times 10^{-7} = \lambda = \frac{(2.9979 \times 10^{8}\, m\, \cancel{s}^{-1})(6.62608 \times 10^{-34}\, \cancel{J}\cancel{s})}{4.9 \times 10^{-19}\, \cancel{J}}$$

$4.05 \times 10^{-7}_{m} / 10^{-9} = \boxed{405.39\, nm \text{ violet light}}$

16 $1\,W = 1\,J\,s^{-1}$ a 10 W laser produces 520 nm light. (5.2×10^{-7} m)

a. Energy Carried by each photon

$$E = hV = hC/\lambda = \frac{(2.9979 \times 10^{8}\, \cancel{m}\, s^{-1})(6.62608 \times 10^{-34}\, J/\cancel{s})}{(5.2 \times 10^{-7}\, \cancel{m})} = \boxed{3.82 \times 10^{-19}\, J}$$

B. # of photons emitted per second.

$$10\, J/s \times \left(\frac{1\,\text{photon}}{3.82 \times 10^{-19}}\right) = \boxed{2.62 \times 10^{19}\, \text{photons}/s}$$

20. Bohr model to Calculate radius and Energy of He^+ in the $n=5$ state.
How much energy is req. to remove 1 mole of e^- from He^+?
What frequency & λ of light is emitted from $n=5 \rightarrow n=3$?

$a_0 = 5.29 \times 10^{-11} M$

Bohr [4.12] $\quad r_n = \frac{n^2}{Z} a_0 \quad a_0 = \frac{\varepsilon_0 h^2}{\pi e^2 m_e}$

$\boxed{r_5}$

$n=5$
$Z=2$

$r_n = \frac{5^2}{2}(5.29 \times 10^{-11} M) = \boxed{6.6125 \times 10^{-10} M}$

$\text{Energy} = -\frac{Z^2}{n^2}(2.18 \times 10^{-18} J) = \boxed{-3.49 \times 10^{-19} J}$

To remove an electron we change $E=0$

(Final) − (initial)
$(0_J) - (-3.49 \times 10^{-19} J) = 3.49 \times 10^{-19} J$ per atom

$\frac{3.49 \times 10^{-19} J}{\text{atom}} \left(\frac{6.022 \times 10^{23} \text{ atoms}}{1 \text{ mole}}\right) = \boxed{\approx 210 \text{ KJ/mol}}$

Energy of $n=3 \quad -\frac{2^2}{3^2}(2.18 \times 10^{-18} J) = -9.688 \times 10^{-19}_{N=3}$ J/atom

(Final) − (initial) =
$-9.69 \times 10^{-19} J - (-3.49 \times 10^{-19} J) = \boxed{-6.2 \times 10^{-19} \text{ J/atom}} = \Delta E$

$\nu = \frac{\Delta E}{h} = \frac{-6.2 \times 10^{-19} J}{6.6261 \times 10^{-34} Js} = 9.357 \times 10^{14} s^{-1} = \nu$

$c = \lambda \nu \quad \frac{2.9979 \times 10^{8} Ms^{-1}}{9.357 \times 10^{14} s^{-1}} = 3.203 \times 10^{-7} M \cdot 10^9 = $ $\boxed{N = 5 \rightarrow 3 \quad 320.3 \text{ nM}}$

24 A green laser ejects no e^- from copper. What about a red laser?

green ~ 532 nM
red ~ 633 nM (as $\lambda\uparrow$ $E\downarrow$)

No Electrons would be ejected.

26 Maximum λ that can be used in an alarm.
Na work function $= 4.41 \times 10^{-19}$ J

$C = \lambda \cdot V$ $\quad C = \dfrac{\lambda \Delta E}{h}$ find λ $\quad \lambda = \dfrac{C h}{\Delta E}$

$V = \dfrac{\Delta E}{h}$

$C = 2.9979 \times 10^{8}\ \mathrm{Ms^{-1}}$
$h = 6.626 \times 10^{-34}\ \mathrm{Js}$
$\Delta E = 4.41 \times 10^{-19}\ \mathrm{J}$

$\boxed{\lambda = 4.504 \times 10^{-7}\ \mathrm{m} = 450.4\ \mathrm{nM}}$

32 Calculate the de Broglie wavelength:
A. e^- accelerated to a KE of 1.20×10^{7} J/Mol
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$KE = T = eV$

$$\lambda = \frac{h}{\sqrt{2MeV}}$$

$$\lambda = \frac{h}{\sqrt{2MT}} = \frac{(6.626 \times 10^{-34}\ \mathrm{Js})}{\sqrt{2(9.109 \times 10^{-31}\ \mathrm{Kg})(1.20 \times 10^{7}\ \mathrm{J\ Mol^{-1}} / 6.022 \times 10^{23}\ \mathrm{mol^{-1}})}}$$

$= 0.11 \times 10^{-9}$ M

T = KE per Particle.

B He moving at $353 ms^{-1}$ He = 4.0026

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\ Js}{\left(\frac{4.0026\ g\ He/mol}{6.022\times10^{26}\ u/kg}\right)(353\ ms^{-1})} = \boxed{0.282\times10^{-9}\ m}$$

C Krypton atom moving at $299 ms^{-1}$

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\ Js}{\left(\frac{83.798\ u}{6.022\times10^{26}\ u/kg^{-1}}\right)(299\ ms^{-1})} = \boxed{0.0159\times10^{-9}\ m}$$

36 Uncertainty in the speed of light is $3\times10^{8}\ ms^{-1}$ (at most)

A. minimum uncertainty in the position of an e^-.

Heisenberg Principle

$$(\Delta x)(\Delta P) \geq h/4\pi$$
$$(\Delta x)(\Delta mv) \geq h/4\pi$$

$m_e = 9.109\times10^{-31}\ kg$
$V = 3.0\times10^{8}\ ms^{-1}$
$h = 6.626\times10^{-34}\ Js$

$1J = \frac{kg\,m^2}{s^2}$

$$\frac{(6.626\times10^{-34}\ J/s)}{4\pi\cdot(9.109\times10^{-31}\ kg)(3.0\times10^{8}\ m/s)} = \boxed{\begin{matrix}1.93\times10^{-13}\ m \\ 0.002\ \text{Å}\end{matrix}}$$

$$\frac{\frac{\cancel{kg}\cdot m^{\cancel{2}}}{\cancel{s^2}}\cdot\cancel{s}}{\cancel{kg}\cdot\cancel{m}/\cancel{s}} = m$$

36 B) Repeat for a He atom

$$\frac{h}{4\pi \cdot M \cdot V} = \frac{6.626\times10^{-34}\,Js}{4\pi \cdot (6.65\times10^{-27}\,Kg)(3.0\times10^{8}\,M/s)} = \boxed{2.64\times10^{-17}\,M}$$

$$He = \frac{4.0026\,u}{6.022\times10^{26}\,Kg/u} = 6.65\times10^{-27}\,Kg$$

38) Absorbed light 800nm

Box of length L

Calculate L

light does excite the e^-

$$\Delta E = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\,Js)(2.9979\times10^{8}\,M/s)}{800\times10^{-9}\,M}$$

$$\Delta E = 2.48\times10^{-19}\,J$$

$$\Delta E = \frac{h^2}{8M_e L^2}\left[(2^2+1^2+1^2)-(1^2+1^2+1^2)\right]$$

$$6-3=3$$

$h = 6.626\times10^{-34}\,Js$

$M_e = 9.109\times10^{-31}\,Kg$

$$L = \sqrt{\frac{h^2\cdot 3}{8M_e\,\Delta E}} = \sqrt{\frac{(6.626\times10^{-34}\,Js)^2\cdot 3}{8\cdot(9.109\times10^{-31}\,Kg)(2.48\times10^{-19}\,J)}} = \boxed{\begin{array}{l}8.53\times10^{-10}\,M \\ 8.53\,\text{Å} = L\end{array}}$$

54 Express velocity of the Bohr model using $M_e, e, h, \mathcal{E}_0, Z$, and N

Eq. 4.12

Eq = 4.13 $$V = \frac{nh}{2\pi M_e r_n} \qquad r_n = \frac{\mathcal{E}_0 n^2 h^2}{\pi Z e^2 M_e}$$

$$V = \frac{\cancel{n}\cancel{h}}{2\cancel{\pi}\cancel{M_e}} \cdot \frac{\cancel{\pi} Z e^2 \cancel{M_e}}{\mathcal{E}_0 n^{\cancel{2}} h^{\cancel{2}}} = \boxed{\frac{Ze^2}{2\mathcal{E}_0 n h} = V}$$

$He^+ = Z = 2$
$U^+ = Z = 92$

$$\left(\frac{Z}{n}\right)\frac{e^2}{2\mathcal{E}_0 h}$$

$e = 1.60218 \times 10^{-19}\,C$
$h = 6.626 \times 10^{-34}\,J/s$
$\mathcal{E} = 8.8542 \times 10^{-12}\,C^2 J^{-1} M^{-1}$

$$\left(\frac{2}{1}\right) \cdot 2187720.606\,M/s = \boxed{4.375 \times 10^6\,M/s \text{ for } He^+}$$

$$\left(\frac{92}{1}\right) \cdot 2187720.606\,M/s = \boxed{2.013 \times 10^8\,M/s \text{ for } U^+}$$

Speed of light $= 2.9979 \times 10^8$

Reletaristic effects matter for U^+ but not for He^+.

60 A photon hase a life of 10^{-10} s find the energy find the frequency

$$(\Delta E)(\Delta t) \geq h/4\pi$$

$$\Delta E \geq \frac{6.626\times10^{-34}\,J\cdot s}{4\pi\,(10^{-10}\,s)} = \boxed{5.27\times10^{-25}\,J} = \Delta E$$

$$V = \frac{\Delta E}{h} = \frac{5.27\times10^{-25}\,J}{(6.626\times10^{-34}\,Js)} = \boxed{7.9\times10^{8}\,s^{-1} = V}$$

HW 4 Other Problems.

Computer Program:
Soduim: Change of the frequency & intensity of radiation.

- As frequency ↑ e^- start to eject from the metal.
- As intensity ↑ the # of e^- ejected ↑.

Changing the Potential of the battery Changes the direction & speed the e^- flow. The e^- are drawn to the "+" plate, and higher voltage increases the speed the e^- flow after ejection.

Workfunction - energy required for an e^- to come off the metal.
e^- start to come off at 538 nM

$c = \lambda \nu$
$\nu = \frac{c}{\lambda}$

$$\frac{3.0\times10^{8}\,Ms^{-1}}{538\times10^{-9}\,M} = 5.76\times10^{14}\,s^{-1} = \nu_0 = \text{minimum frequency to eject } e^-$$

$\nu_0 = 5.76\times10^{14}\,s^{-1}$

$h = 6.626\times10^{-34}\,Js$

$$\boxed{\Phi = h\nu_0 = 3.81\times10^{-19}\,J \text{ for Na}}$$

Copper e^- come off at $\approx$ 259 nM

$$\nu = \frac{c}{\lambda} = \frac{3.0\times10^{8}\,ms^{-1}}{259\times10^{-9}\,M} = 1.158\times10^{15}\,s^{-1}$$

$$\Phi = h\nu_0 = (1.158\times10^{15}\,s^{-1})(6.626\times10^{-34}\,Js) =$$

$$\boxed{\Phi = 7.67\times10^{-19}\,J}$$

Copper has a higher work function.

Electrons are more tightly bound to Cu than Na, following the trend of Ionization energy. this makes sense,

$E = Mc^2$ $E = h\nu$ show that momentum $(Mc) = h/\lambda$ $c = \lambda \cdot \nu$

$$E = M \cdot c \cdot c = h\nu$$

$$E = M \cdot c \cdot \lambda \cdot \nu = h\nu$$

$$\boxed{E = M \cdot c = \frac{h\cancel{\nu}}{\lambda\cancel{\nu}} = \frac{h}{\lambda}}$$

1 Probability of finding the particle in the box between $X = 0$ and $X = L/2$ for $n = 1$

$$\int_0^{L/2} \left(\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi X}{L}\right)\right)^2$$

$$\int_0^{L/2} \frac{2}{L} \sin^2\left(\frac{n\pi X}{L}\right) \rightarrow \frac{2}{L} \int_0^{L/2} \sin^2\left(\frac{n\pi X}{L}\right)$$

(11)

$$\frac{2}{L}\int_0^{L/2} \frac{1}{2}\left(1-\cos\left(\frac{n\pi x}{L}\right)\right)$$

$$\frac{1}{L}\int_0^{L/2}\left(1-\cos\left(\frac{2n\pi x}{L}\right)\right)dx$$

$$\frac{1}{L}\int_0^{L/2} 1dx - \int_0^{L/2}\cos\left(\frac{2n\pi x}{L}\right)dx$$

$$u=\frac{2n\pi x}{L}$$

$$\frac{1}{L}\int_0^{L/2}dx - \frac{L}{2n}\cdot\frac{1}{L}\int_0^{L/2}\cos u\,du$$

$$\frac{1}{L}\left[x\right]_0^{L/2} - \frac{1}{2n}\left[\sin\left(\frac{2\pi n}{L}\right)\right]_0^{L/2}$$

$\frac{1}{2} = \frac{1}{L}\cdot\frac{L}{2}$

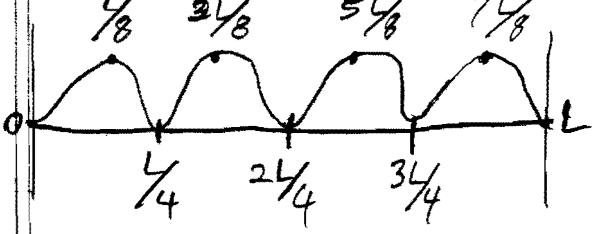
$$\boxed{= \frac{1}{2} \text{ or } 50\% \quad \text{for } n=1}$$

for $n=3$ the probability is still $\frac{1}{2}$ or 50%

(2) for the $n = 4$ state at what distance is the probability of finding the particle the largest?

At what distance are the nodes?

$N=4$

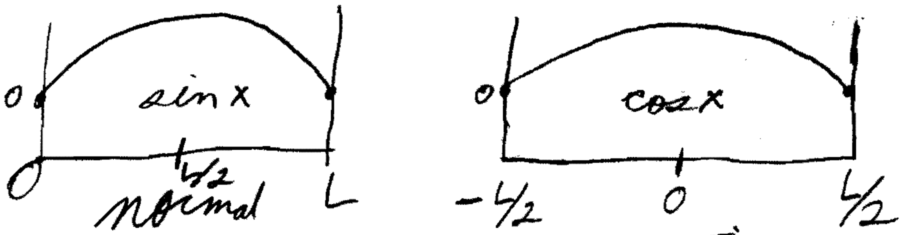


Nodes: $L/4,\ L/2,\ 3L/4$

Greatest Probabi.

$L/8,\ 3L/8,\ 5L/8,\ 7L/8$

(3)



Redefined. (12)

Boundary condition must be satisfied so Ψ would change

(4) Zero Point energy of a 3-D particle in a box.

$L_x = 1\text{Å}$, $L_y = 2\text{Å}$, $L_z = 1\text{Å}$

$$E_{xyz} = \frac{h^2}{8M}\left[\frac{n_x^2}{L_x} + \frac{n_y^2}{L_y} + \frac{n_z^2}{L_z}\right]$$

$n = 1$

$M =$ Mass of e^-

$M = 9.109 \times 10^{-31}$ Kg

$$\frac{(6.626 \times 10^{-34}\ Js)^2}{8 \cdot (9.109 \times 10^{-31}\ Kg)}\left[\frac{1^2}{(1 \times 10^{-10}\ m)^2} + \frac{1^2}{(2 \times 10^{-10}\ m)^2} + \frac{1^2}{(1 \times 10^{-10}\ m)^2}\right]$$

$$\left[2.25 \times 10^{20}\ \frac{1}{m^2}\right]$$

$$= \boxed{1.355 \times 10^{-17}\ J}$$

$\frac{h^2}{8ML^2}[n_x^2 + n_y^2 + n_z^2]$

(5) lowest energy state $= [1^2 + 1^2 + 1^2]C = 3C$

set $\frac{h^2}{8ML^2}$ = Constant $= C$

$18 - 2 = 16$

$3C \times 6 = 18C$

$[1^2 + 1^2 + 4^2] = 18$

$(1,1,4); (1,4,1); (4,1,1)$ $\boxed{\text{degeneracy } 3}$